

## AN INVESTIGATION OF ALTERNATIVE TOLL FACILITY ASSIGNMENT METHODS

In this paper, we provide a conceptual and empirical comparison of traffic assignment methods that are or can be used to forecast toll road utilization. Our focus is primarily on static equilibrium methods and static toll levels, but we also present a dynamic model and compare its results with static models. For dynamic pricing strategies, we also explore the application of dynamic solutions which we show to be necessary for valid analysis of these measures. In addition to methodological comparisons, we illustrate the fundamental point that the distribution of value of time among travelers is the key determinant of toll facility utilization.

### **Current Traffic Assignment Methods and Practices**

There are several basic approaches to equilibrium toll road assignment commonly encountered in practice. Some form of equilibrium assignment has become recognized as essential because of the need to achieve consistency in model inputs and outputs be these solely in the traffic assignment model or in a model sequence that culminates with a traffic assignment and has feedback to earlier stages of the model possibly to distribution or mode choice or both of these model components. The time of day of travel may also be influenced by travel times and costs that differ by time of day.

We focus here on the traffic assignment aspects and extensions. Mode choice, time-of-day of travel, and other more advanced demand considerations are discussed in NCHRP 722 Vol.2 and all over the research literature. Of course most of these other model components would be impaired by faulty traffic assignments and incorrect congested travel times.

The basic variation introduced by accounting for toll roads in traffic assignment is that a price is to be paid for a faster and potentially more reliable route. This price or toll may vary by vehicle class and classes may differ in the value they place on time in different ways than are reflected in the toll. Tolls may be link-based such as a toll for a bridge crossing or may be more complex as in the case of exit-to-entrance tolls that cannot be represented correctly by link tolls.

Link-based assignment algorithms can account for ramp-to-ramp or entrance-to-exit tolls, but not without some methodological extension and this feature, which is in TransCAD, is not supported in all other packages. Note that entrance-to-exit tolls are more varied than O-D tolls because different routes for a given O-D pair can use different toll road entrance and exit ramps.

Standard, good practice for toll road modeling is to use a multi-class assignment that respects link exclusions, represents user classes with different mean values of time, and accurately represents the tolls that are applicable. This method is recommended in NCHRP 722, although it is an open question how many different user classes should be used.

A different method that is in use, particularly in “investment grade” studies and also in road pricing studies, is a legacy method that is based upon computing the best toll path and the best non-toll path at each iteration of the assignment calculation and applying a logit model to split the O-D traffic loaded between the two paths based on the toll and possibly other factors. As we will see later, this method is flawed and should no longer be used.

More advanced and effective methods consider value of time distributions explicitly, but have been rarely used in practice. Dial introduced a bi-criterion traffic assignment he called T2 in which both time and cost or another dimension could be volume-dependent. When applied to toll road assignment, a stochastic, value of time distribution is used.

Leurent proposed a time and cost assignment with a value of time distribution but with only time being volume-dependent. Caliper successfully implemented this method for SETRA, the French Ministry of Transport where it is used for modeling toll roads. Leurent's method is solved by the method of successive averages (MSA).

Dial's T2 algorithm is more ambitious and, in his work, was solved with both Frank-Wolfe and simplicial decomposition algorithms, with the latter being considerably faster than the former. The choice of solution method is of consequence as T2 is computationally demanding. Dial also introduced an extremely efficient means of evolving shortest path trees in conjunction with value of time progressions.

A bicriteria time and cost assignment is a more attractive solution than other toll road methods employed in current planning practice. This method recognizes that values of time in terms of toll sensitivity vary throughout any single class of travelers. It can be shown that approximating this distribution with a small, but finite number of classes each with its own "average" value of time leads to distorted results and toll revenue estimates.

Recognizing the potential of the T2 model, Caliper has implemented it within the context of our multi-modal, multi-class master assignment procedure. A multi-modal, multi-class method is needed to take account of network exclusions for certain classes of traffic, to deal with varying passenger car equivalents for trucks and buses, ramp-to-ramp tolls in addition to link tolls, and to take account of toll rates that vary by vehicle class.

In working with T2, we have also experimented with different UE solution algorithms. Specifically, we have implemented an Frank-Wolfe (FW), simplicial decomposition (SD), a biconjugate Frank-Wolfe BFW, and a path-based OUE solution based upon Dial's algorithm B. Each of these implementations was multi-threaded for good computational performance and to achieve the high levels of convergence that are necessary to assess project impacts accurately.

### **A Simple Illustrative Example of Equilibrium Toll Road Assignment Methods**

To see the extreme differences in results that are associated with different traffic assignment approaches for toll roads, we will present a small hypothetical example for a single origin-destination pair. The network is shown in Figure 1. There are two toll links that provide short free flow travel times of 10 minutes and other links that have free flow travel times of 20 minutes each. As shown in Figure 2, the first and last links have capacities of 200 vehicles per hour which will not restrict the flow of the 100 trips that will be assigned. The interior links all have capacities of 50 vehicles per hour.

To reach the destination from the origin, the links in this network can be traversed in three ways. In the first, no toll links are utilized. In the second path, only the less expensive toll link is utilized, and in the third path, both toll links are traversed.

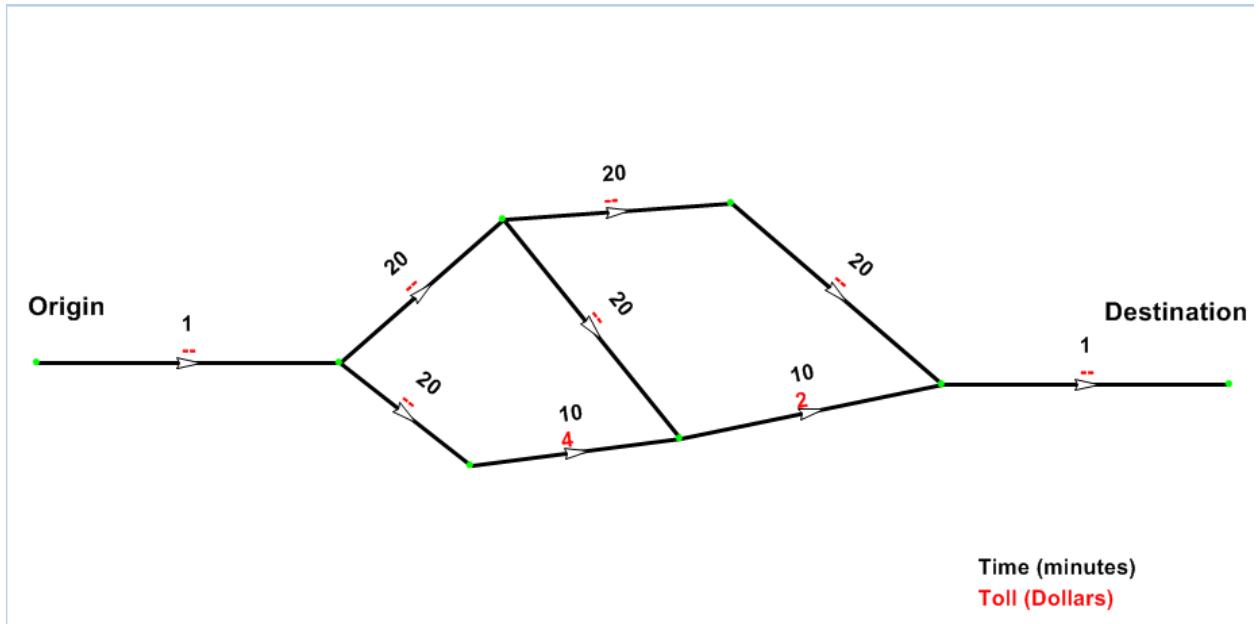
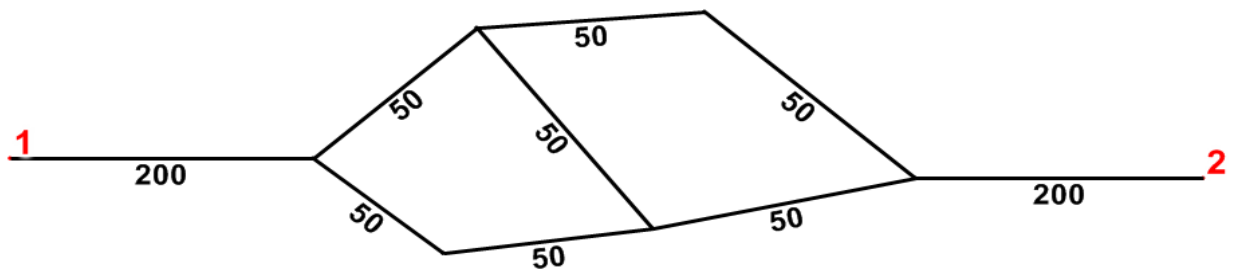


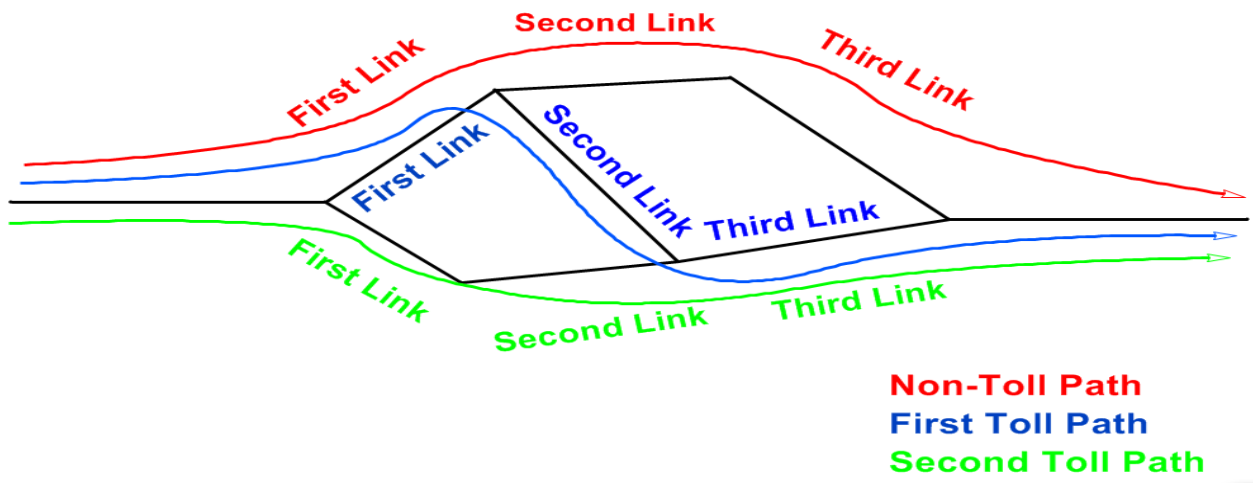
Figure 1 Network Travel Times and Tolls

	1	2
1	0.00	100.00
2	0.00	0.00

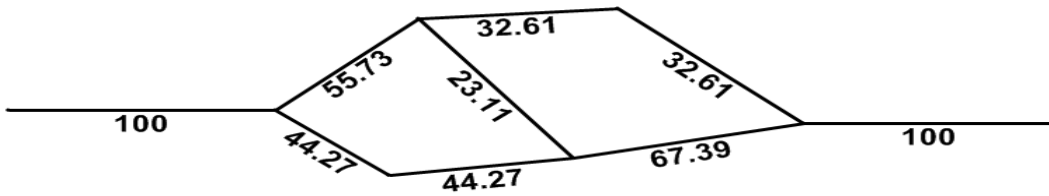


## Link Capacity

Figure 2 Link Capacities and O-D Demand



## Final Iteration Solution



Single class UE with VOT of 20/per hour

In the table below, we summarize the results of the assignment in terms of path times and other output measures such as VHT and toll revenue.

	STD. UE GENERALIZED COST ASSIGNMENT			
	non-toll path	first toll path	second toll path	
	travel time	travel time	travel time	
first link	24.63	24.63	21.84	
second link	20.54	20.14	10.92	toll
third link	20.54	14.95	14.95	links
total time	65.71	59.72	47.71	
Vol	32.61	23.11	44.27	
Toll	0	2	6	
time value of toll	0	6	18	
generalized cost in tt at \$20/hr. VOT	65.71	65.72	65.71	
toll revenue	0	46.22	381.42	427.64
VHT	5635.054			

At user equilibrium, all used paths have the same generalized cost. One can see that the equilibrium condition is met when as the generalized cost is the same 65.7 minutes for each path.

In contrast, we perform a toll diversion assignment on the same network using the same \$20/hour value of time. The specific logit equation is given below:

- $U_{Toll} = C + \alpha_1(Time_{Toll} - Time_{Free}) + \alpha_2(Cost_{Toll} - Cost_{Free}) + \alpha_3(Dist_{Toll} - Dist_{Free})$
- $P_{Toll} = \frac{1}{1 + e^{-U_{Toll}}}$

In the example above,

- $C = 0$
- $\alpha_1 = -1.0$
- $\alpha_2 = -3.0$
- $\alpha_3 = 0$
- $VOT = \frac{\alpha_1}{\alpha_2} = 0.33\$/min = \$20/hr$

In the toll diversion method, toll values are not used in finding the best toll path which is a problem when there can be more than one toll path for a single origin-destination pair. Typically, the toll values also are not part of the relative gap calculation used as a stopping criterion for the equilibrium calculation.

The results for the toll diversion assignment are shown below.

	TOLL DIVERSION ASSIGNMENT			toll links
	non-toll path travel time	first toll path travel time	second toll path travel time	
first link	20.85	20.85	27.84	
second link	20.57	20	13.92	
third link	20.57	14.83	14.83	
total time	61.99	55.68	56.59	
Vol	33.03	3.4	63.57	
Toll	0	2	6	
time value of toll	0	6 minutes	18 minutes	
generalized cost in tt at \$20/hr. VOT	61.99	61.68	81.57	
toll revenue	0	6.8	381.42	388.22
VHT	5834.268			

In this assignment, only 3.4 trips are assigned to the first toll path which seems unrealistic. As can be observed from these results, a user equilibrium solution is not achieved, as evidenced by the radically different generalized costs associated with the different paths. Moreover the highest volume path has the highest generalized cost and the path with the lowest generalized cost attracts the fewest trips. Clearly, the Wardrop equilibrium condition that all used paths have the same minimum generalized cost is not satisfied because travelers could shift from the third path to either of the other two and experience lower generalized costs.

Note too, how different the estimated VHT and toll revenue are when compared to the standard UE assignment. VHT is over-estimated and toll revenue is under-estimated in this example. For other examples, different behaviors might be observed depending upon free-flow speeds and toll levels.

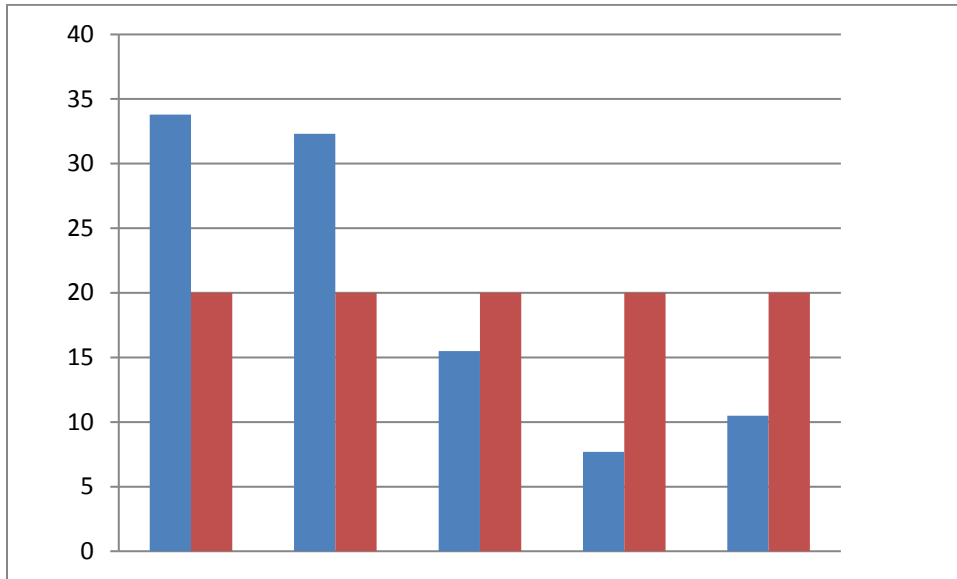
Practitioners of the toll diversion method may often use constants to control the share of toll road users by O-D pair. But when there is only one O-D pair, that cannot be done and the shortcomings of the method are revealed.

While the regular UE solution is clearly superior to the toll-diversion method, it is also a simplification of reality in that different travelers have different values of time. The effects can be seen from computing a multi-class traffic assignment in which the classes are segmented by different values of time. First, we assume a uniform distribution of vot with 5 classes and the same mean vot of \$20/hour. The classes have vots of 0, 10,20, 30, and 40 dollars per hour, respectively.

	Uniform non-toll path travel time	STD. MMA UE GENERALIZED COST ASSIGNMENT 0,10,20,30,40 VOT classes	
		first toll path travel time	second toll path travel time
first link	24.69	24.69	21.81
second link	21.23	20.03	10.91
third link	21.23	13.1	13.1
total time	67.15	57.82	45.82
Vol	40	15.91	44.08
Toll	0	2	6
mean time value of toll	0	6	18
generalized cost in tt at \$20/hr. VOT	67.15	63.82	63.82
toll revenue	0	31.82	264.48
VHT	5625.6618		296.3

Here we have slightly lower vht and aslo lower toll revenue than for the single class UE, presumably because there more travelers with low values of time.

A more realistic value of time distribution is lognormal. For purposes of illustration, we use a lognormal distribution with same mean value of time of \$20. The percentage of travelers by value of time is shown in the graph below and compared to the uniform distribution. It has a wider dispersion in that there are some travellers with a higher value of time than \$40 per hour but also many more with low values of time.



The mma assignment with the same number of classes but with the lognormal distribution of class volumes by value of time is shown below.

	Lognormal non-toll path travel time	STD. MMA UE GENERALIZED COST ASSIGNMENT 0,10,20,30,40 VOT classes		
		first toll path travel time	second toll path travel time	
first link	29.19	29.19	20.63	
second link	22.09	20.08	10.32	toll links
third link	22.09	12.09	12.09	
total time	73.37	61.36	43.04	
Volume	45.67	20.47	33.86	
toll	0	2	6	
mean time value of toll	0	6	18	
generalized cost in tt at \$20/hr. VOT	73.37	67.36	61.04	
toll revenue	0	40.94	203.16	244.1
VHT	6064.1815			



In the table above, we observe higher VHT and lower toll revenue than in the original assignment with a single user class and the same mean value of time. This may explain some of the optimism bias in analyses of toll roads.

As a last example with this notional network, we run a T2 assignment with the same value of time distribution. The difference is that the value of time is not represented with five classes but rather with a discrete approximation to a continuous distribution. In this case we used a 20bin histogram.

	lognormal non-toll path travel time	T2 OUE - Resolution = 20 Average VOT = \$20/hr, Std. Dev = \$20/hr		toll links
		first toll path travel time	second toll path travel time	
first link	30.38	30.38	20.49	
second link	21.52	20.22	10.25	
third link	21.52	12.68	12.68	
total time	73.42	63.28	43.42	
vol	42.18	26.01	31.81	
toll	0	2	6	
mean time value of toll	0	6	18	
generalized cost in tt at \$20/hr. VOT	73.42	69.28	61.42	
toll revenue	0	52.02	190.86	242.88
VHT	6123.9586			

This result gives slightly lower toll revenue and slightly higher vht due to fewer travelers using the entire toll facility.

## Simple network with more realistic flows

To show that the notional example is valid, we run a similar network with more realistic flows and capacities. In this example, the toll road has a nominal capacity of 1800 per lane per hour and the non-toll highway has a capacity of 1400 per hour per lane. All roads have two lanes.

We load the network with 5800 vehicles in a one hour period and consider the same cases as above in terms of tolls, assignment formulations, and values of time. The first example is the standard single class, user equilibrium assignment with a VOT of \$20/hr.

	STD. UE GENERALIZED COST ASSIGNMENT			
	non-toll path	first toll path	second toll path	
	travel time	travel time	travel time	
first link	18.06	18.06		
second link	25.31	12.27	18.33	toll links
third link		7.04	7.04	
total time	43.37	37.37	25.37	
TransCAD Volume	2116	1666	2019	
toll	0	2	6	
time value of toll	0	6	18	
generalized cost in tt at \$20/hr. VOT	43.37	43.37	43.37	
toll revenue	0	\$3,332	\$12,114	<b>\$15,446</b>
VHT	205,251			

As in the case above, the user equilibrium condition is achieved with same generalized cost obtained for each path. In contrast the toll diversion assignment gives peculiar results.

	TOLL DIVERSION ASSIGNMENT			
	non-toll path	first toll path	second toll path	
	travel time	travel time	travel time	
first link	12.07	12.07		
second link	24.18	12.05	27.02	toll links
third link		9.04	9.04	
total time	36.25	33.16	36.06	
TransCAD Volume	945	1	4854	
toll	0	2	6	
time value of toll	0	6	18	
generalized cost in tt at \$20/hr. VOT	36.25	39.16	54.06	
toll revenue	0	\$2	\$29,124	<b>\$29,126</b>
VHT	209,325			

Only one trip is assigned to the first toll path and we observe the similar pattern in that the toll diversion overstates both the toll revenue and vht.

Next we show the results for the two MMA assignments with differing value of time class distributions.

	uniform	STD. MMA UE GENERALIZED COST ASSIGNMENT 0,10,20,30,40 VOT classes		
	non-toll path	first toll path	second toll path	
	travel time	travel time	travel time	
first link	16.36	16.36		
second link	25.83	12.1	18.53	toll links
third link		6.83	6.83	
total time	42.19	35.29	25.36	
TransCAD Volume	2320	1160	2320	
TransModeler Volume	1172	2346	2282	
toll	0	2	6	
mean time value of toll	0	6	18	
generalized cost in tt at \$20/hr. VOT	42.19	41.29	43.36	
toll revenue	0	\$2,320	\$13,920	<b>\$16,240</b>
VHT	197,652			

	lognormal	STD. MMA UE GENERALIZED COST ASSIGNMENT 0,10,20,30,40 VOT classes		
	non-toll path	first toll path	second toll path	
	travel time	travel time	travel time	
first link	18.41	18.41		
second link	30.34	12.05	18.3	toll links
third link		6.29	6.29	
total time	48.75	36.75	24.59	
TransCAD Volume	3204	631	1964	
toll	0	2	6	
mean time value of toll	0	6	18	

generalized cost in tt at \$20/hr. VOT	48.75	42.75	42.59	
toll revenue	0	\$1,262	\$11,784	<b>\$13,046</b>
VHT	227,679			

	T2 OUE - Resolution = 20			
	lognormal non-toll path travel time	Average VOT = \$20/hr, Std. Dev = \$20/hr first toll path travel time	second toll path travel time	
first link	19	19		
second link	31.45	12.05	18.26	toll links
third link		6.24	6.24	
total time	50.45	37.29	24.5	
vol	3339	583	1878	
toll	0	2	6	
mean time value of toll	0	6	18	
generalized cost in tt at \$20/hr. VOT	50.45	43.29	42.5	
toll revenue	0	1166	11268	<b>\$12,434</b>
VHT	236,203.62			

What we have established is that the shape of the distribution of the value of time has a pivotal impact on predicted toll road utilization and revenue. This is both obvious and logical and puts the onus on obtaining input data from stated preference studies and other surveys of toll road use.

T2 uses a histogram with more points the more accurate the answer. This is more efficient than having separate user classes for each vot with more and more shortest path trees that would be required.

Los for the output

Convergence issues do not arise in this simple example as any method will converge on this network with a single O\_D pair. Of course if the wrong gap measure is used the convergence will be to an incorrect solution. In more complex examples, the choice of algorithm and the level of convergence will most definitely have an impact on the toll road utilization estimates.

A More Realistic Case

A more realistic case involves multiple origins and destinations and an existing or proposed toll facility. For this we examine a subarea of Miami FL in which Thesedata were supplied by Florida DOT

Computing time and memory are also obviously not issues.

In the simulation weaving to effectuate the choice of toll road or non-toll road use actually causes queuing backups from the diverge point.

Los for the links.

If the tolls are too high, then an elastic demand formulation would be necessary because of trip suppression

What about toll diversion with tolls in the toll path...still has the problem of multiple paths

There can be a toll-road exclusion class to preserve the mode choice results...for an o-d pair. Why did vovsha say not?

Providing for a limited number of penalties would enhance the BFW more than anything else.

Income isn't the right way to segment by vot class

Side constraints or at least validation by class counts or toll versus non-toll counts in the base case  
Gap measure discussion perhaps for the Miami example...  
How different is toll diversion in the simulation...

If toll roads do not suppress overall travel, they are a negative for air quality.

Talk about more complex tolling practices and price schedules.

Show SUE with msa too?

Can we warm start t2 with the efficient frontier estimate?

Vary demand parametrically

Optimal tolls through grid search...and different methods...

The mn stuff is not based upon anticipating future conditions either from history or by forecasting. Duh!

Conclusions

Toll revenue estimates are highly sensitive to the value of time distribution and also to the toll road assignment methodology.

It appears that toll revenues are inversely correlated with vehicle hours of travel so successful toll roads increase congestion overall rather than relieving it.

In t2, both criterion can be volume dependent so we can control the v/c ratio on the toll links to be .9 or .8 or can we?

How do we do this...the toll must be pegged to the average VOT or the median or mode...

Algorithms for UE assignment—FW, BFW, and path-based OUE

Convergence levels are relevant

We could use the gw bridge toll example....

Where are there entrance to exit tolls in the u.s.? certainly the ohio turnpike,

We can use our data to quantify the error

Representation of tolls

A variety of tolling schemes are utilized at present.

Preferential pricing is given to transponder holders...

Other types of pricing schemes and their treatment in traffic assignment-corridor and cordon pricing,  
zone

dynamic models

General issues with multi-class assignment

What does the select link analysis show from the cube on the hot lane project

Toll booth queuing less important given the widespread use of electronic payment methods that may not even require slowing down.

Dynamic pricing problems pose particular challenges for traffic assignment and not surprisingly require dynamic models.

Attempts to model dynamics with static models could possibly work, but approaches encountered in practice do not.

Hot lanes exemplify entrance to exit tolls,

The dynamic model provides a useful point of reference against which to judge simpler methods.

Bi-criterion models can have flow-dependent times and prices....

Miami Experiments

A comparative analysis can reveal the errors associated with various models of the same scenario. Hypothesize that the microsimulation is the reality and compute its characteristics for input into less detailed simulation and traffic assignment model formulations. This entails deriving 15 minute counts, speeds, VOTs, and O-D matrices with which to test the alternative models for HOT lanes.

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