A NEW TRAFFIC ASSIGNMENT METHOD FOR SMALL AND MEDIUM COMMUNITIES

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August 6, 2010

ABSTRACT

This paper describes a new equilibrium traffic assignment method, compares it with prior methods, and illustrates how it can provide improved results for the models that are applied in small and medium-sized communities. The new origin user equilibrium (OUE) method, based on an algorithm developed by Robert Dial, provides superior convergence and more reliable estimates of link flows than the conventional method provided in most travel demand forecasting software packages.

Utilizing a regional model for Victoria, British Columbia, we compare traffic assignment results for test cases at different levels of assignment convergence and using different algorithms for computing user equilibrium. We demonstrate that OUE can achieve levels of convergence that are virtually indistinguishable from the true equilibrium solution and do so rather quickly on commonly available computers.

The test cases illustrate that low levels of convergence lead to gross errors and spurious effects in traffic assignment, but that these problems disappear with higher levels of convergence. Consequently, the ability of OUE to achieve high levels of convergence quickly leads to much more reasonable and reliable traffic assignment results.

Small community planners often rely on select link and select zone analysis to understand who benefits from road improvement projects and to share that information with stakeholders. We examine the select link analysis that is derived from different traffic assignment algorithms and convergence levels. We show that the results are very sensitive to these factors and that, with a proportionality correction, the OUE method provides more robust and reliable select link analysis.

Key words: traffic assignment, impact analysis, select link analysis
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In this paper, we introduce a new equilibrium assignment tool that can dramatically improve traffic forecasting in small and medium size communities. This model is readily deployable and provides more rapidly convergent and accurate static user equilibrium traffic assignments resulting in more dependable select link analysis and impact assessments.

The user equilibrium (UE) concept in traffic assignment is an attractive modeling assumption in that it reflects 1) the volume dependence of road network performance, 2) the natural tendencies of users to seek their best routes and 3) the need for consistency between the travel times used in modeling trip distribution and those generated by the model in the traffic assignment stage. The UE paradigm is widely accepted as a best practice in static (i.e., one period) models and is equally attractive and for the same reasons in dynamic models including stochastic traffic simulation models.

UE can be calculated using different computational methods and to different levels of convergence. Ideally, high convergence would be achieved, although in the past this has generally been discouraged by inadequate algorithms and long computing times. New methods and improved hardware now remove these barriers in static models and computational advances mitigate them for dynamic models.

OUE - A Key New Development in Static User Equilibrium Traffic Assignment

A few years ago, Caliper introduced a new traffic assignment method for computing the standard UE solution sought in most regional models based on theoretical work by Robert Dial. This method is superior to prior and other emerging methods in a variety of ways and has been deployed in a sufficient number of regional models to establish its utility.

The origin user equilibrium (OUE) method computes an equilibrium solution for each origin in addition to an overall link flow equilibrium for the entire the network. OUE converges rapidly and does so to much tighter tolerances than the conventional Frank-Wolfe (FW) method used in most planning software. Also, link flows associated with poor paths that might be generated in early iterations are dropped yielding greater accuracy. The results in a sub-network of utilized links that can be saved for each origin-destination pair and be used to warm start the assignment process. Detailed discussion of this method and key results can be found in Slavin et al. (2009), Dial (2006), and Slavin et al. (2006).

Most good practice models will make use of a multi-class user equilibrium traffic assignment model. Multi-class models can accommodate trucks of different sizes as well as other classifications of travelers or trip purposes and can accommodate toll roads, HOV lanes, or class prohibitions should those be present. For twenty years, the workhouse for calculating user equilibrium in TransCAD has been our own, very fast implementation of the FW algorithm. Several years ago, we multi-threaded this algorithm, resulting in a nearly proportional speedup in computation that is a function of the number of physical cores in a computer’s CPU. Recently, we implemented a further speedup in the form of the bi-conjugate descent FW method (BFW) proposed by Daneva and Lindberg (2003). The BFW method uses a little more memory than the conventional FW assignment, but not so much that it would typically be a concern today. FW holds two link flow vectors in memory where conjugate descent methods keep 3 or more link flow vectors in memory that are used in choosing a more effective search direction than FW. Conjugate descent methods are easily multi-threaded, so there is there is no tradeoff in using them.

For comparison purposes, Figure 1 illustrates the speed of convergence for a model developed in TransCAD for Victoria, British Columbia, a medium size city. The convergence measure used is the relative gap (RG), a common and reasonable figure of merit (Rose et al., 1988). The times reported are for a two-class assignment in seconds on a 2.4GHz Core duo laptop computer.
Figure 1: Convergence graph for the Victoria MMA assignment

As can be seen in the Figure, the convergence rate for OUE is greatly superior to that for FW and BFW. You can also see that the FW assignment method’s rate of convergence tails at a certain point, an observation well-known to modelers. This means that this method will not be able to achieve orders of magnitude lower convergence. BFW is much faster in achieving modest levels of convergence than FW, but BFW also tails albeit at an order of magnitude lower gap. You may find it interesting to know that it takes the FW method approximately 250 and 1000 iterations to reach relative gaps of .001 and .0001, respectively. In contrast, the OUE method continues to converge rapidly and to unprecedented levels of convergence. In five minutes, OUE gets to a relative gap of nearly $10^{-8}$. The ability to calculate to that level and beyond makes it possible to quantify the degree of convergence error in link flows from less converged assignments.

Most models in the U.S. have traditionally used a relative gap of .01, but that is insufficient for impact assessment, and as this has become more widely known, increasingly tighter assignments are being computed. Nevertheless, gaps below .001 are rarely encountered for regional models due to slowly convergent algorithms or insensitivity to the issue.

Not only does OUE permit the computation of highly converged traffic assignments, but it has further advantages to which we now turn.
Immediate Availability of Select Link Analysis as a Post Process

When the results are saved after the computation of an OUE assignment, select link analysis can be performed for any query as a post-process. This means you do not have to specify the query before running the assignment, and you can save a great deal of time by not having to run another traffic assignment when another query is desired.

Warm Start Speedups with OUE

A particularly important aspect of OUE is that it can re-compute a new equilibrium from a prior solution, even if the trip table or network attributes have changed somewhat. Prior solutions are nearly always available because traffic assignments are computed over and over again in the course of model development. This is very helpful in a model with feedback loops where the trip tables change at each iteration while the network remains fixed (Slavin et al., 2007).

Removing Errors in Assignment Link Flows

Research that we have performed continues to establish that that poor convergence leads to large errors in traffic assignments and, thus, in all aspects of planning models and impact assessments. (Slavin et al, 2009, Boyce et al, 2004). In particular, poor convergence accounts for the implausible changes in link flows that are often observed in traffic assignments at locations that are remote from the specific projects being evaluated. This can be seen from the following example in which we made a small and substantially irrelevant change to the Victoria network. As shown below, the change was made by disabling one of the links in Sooke, a rural region on the western end of the network. We then reran the assignment and compared it with the base case assignment for the AM period.

![Location of the Irrelevant Change](image)

**Figure 2: Location of the Irrelevant Change**

While the change should only affect links flows in the immediate vicinity of the change, when run to a gap of 0.01 or even .001, there are spurious effects far away. In the maps, only links where the absolute difference is greater than 50 vehicles are shown. Red links indicate those where flow has decreased (in the scenario) and the green links indicate those where flow has increased.
In fact, as shown in Figure 3, changes can be seen as far east as Victoria which is 18 miles away from the project area. The number of links where the absolute flow difference exceeds 50 vehicles is 177. At a gap of 0.001, the number of links where the absolute flow difference was more than 50 drops to 30, yet significant changes can be seen far away from the project area as shown above.

At a gap of 0.0001 however, the random effects disappear and the only regions affected are those in the immediate vicinity of the change. The following figure shows this. Only 13 links have an absolute flow difference of 50 vehicles or more.

A close up of the change location in Figure 4 above shows the localized assignment differences at a gap of 0.0001. These changes are entirely logical re-routings to avoid the disabled link. At this relative gap, there are no remote effects of disabling the link.
Importance of Convergence for Impact Analysis

The elimination of spurious effects is particularly important when performing analysis of the impacts of a specific road improvement project. In the example below, we added a lane to a segment of the TransCanada Highway (CA 1) from Helmcken Road and McKenzie Avenue, which is an essential component of the regional network and one that is scheduled for this expansion in the future. We then re-ran the assignment and examined the changes when compared to the base case.

Figure 5: FW Flow Changes from Added Lanes at a Relative Gap of .001
One can clearly see that the project impacts are quite different in Figures 5 and 6. The differences are attributable both to the different assignment methods and the different levels of convergence. You can note that the results from the FW assignment are smeared and less coherent from the more tightly converged OUE assignment. Quite obviously, the spatial patterns of traffic are also different which would lead to different conclusions the traffic impacts of this scenario. As we have found in many other situations, there is geographic bias associated with poorly converged assignments.

While more research will be needed to establish general guidelines, evidence accumulated to date by us suggests that a relative gap of .0001 or better should be used for accurate results (Slavin et al., 2009.; Boyce et al., 2004) This guidance is likely to be network and project dependent. The appropriate level of convergence required can be tested for specific models rather easily by computing OUE to high convergence and then assessing the link flow errors associated with lesser convergence.

The tests that we have conducted illustrate clearly that estimates of project impacts vary substantially with the convergence level utilized in the traffic assignment. The link flow errors are not randomly distributed, but will typically reflect a particular geographic bias associated with forecast use of some routes instead
of others. The bottom line is that achieving good convergence in the traffic assignment is essential for obtaining valid estimates of project impacts.

**Most Likely Route Flows, Proportionality, and Select Link Analysis**

The user equilibrium assignment computes a set of unique link flows if it is carried out properly and to a small enough relative gap. The route flows associated with any particular equilibrium assignment are not unique, despite the fact that estimates of these route flows are used when performing critical link analysis. The particular route flows generated from a traffic assignment may also be biased because of the method that is utilized for achieving equilibrium, especially if the algorithm’s output depends upon the order in which the different O-D pairs are processed.

If there are multiple possible route flow solutions for a given set of equilibrium link flows, it becomes natural to try to identify the most likely route utilization. The maximum entropy or most likely set of route flows provides an attractive solution to this problem (Bar-Gera, 2010; Boyce et al., 2010). This was suggested by Tom Rossi in his Master’s thesis more than twenty years ago (Rossi, 1987), but we have provided the first readily applicable commercial solution for calculating the most likely route flows. This was done by adding a route flow proportionality correction to the OUE procedure. Fortunately, this takes only small amount of extra computing time.

Select link analysis is sensitive to the level of convergence and also to the algorithmic method used to generate the equilibrium solution. This is easily seen from the maps in Figures 7 and 8 below. The maps show a select link analysis for the same portion of the TransCanada Highway (CA 1) but without the lane additions that were discussed previously.

![Figure 7: Select Link Analysis with FW and a Relative Gap of .001](image-url)
From the Figures above, it can be seen that even at low convergence FW overstates the geographic dispersion of the trips that used the selected links. If the FW solution were computed to a lower gap, even more links would appear in the select link analysis although they would probably have only extremely low fractional flows.

In contrast, the select link results from OUE and greater convergence are more compact and more plausible. They will be more dependable and defensible when presenting traffic assignment results to the public.

**Conclusion**

The ability to calculate user equilibrium traffic assignments with very tight convergence levels provides a lens that permits inspection of many issues associated with convergence. We have illustrated that the results in terms of link flows will vary considerably and that evaluation of project impacts can be improved with better assignment models and tighter convergence. In fact, it is becoming increasingly clear that orders of magnitude improvement in assignment convergence levels are necessary to obtain good estimates of project impacts from planning models. The results from the Victoria examples underscore those that we and others have obtained from performing similar experiments with large regional models all across the country.

In our tests, we have found the OUE traffic assignment is superior to other methods in terms of convergence behavior and will lead to reduced errors in planning models. It has all the features needed for modeling multiple user classes, HOV lanes, toll roads and vehicle restrictions, so it is readily
deployable for existing models. It also makes it straightforward to perform tests that will determine the level of convergence that is appropriate for any model or impact assessment.

With the proportionality correction, OUE provides vastly improved select link results. The improvement stems not just from greater convergence, but also from the fact that, in an OUE assignment, poor paths are completely eliminated in subsequent iterations. While one must remember that select link analysis is still dependent on many modeling assumptions, many of the spurious and difficult to explain effects can be eliminated.
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